Chapter9 Volume and Surface Area



HOW is shape important when measuring a figure?



Content Standards MCC6.G.2, MCC6.G.4

Mathematical Practices 1, 2, 3, 4, 5, 6, 7, 8



Math in the Real World

Aquariums Two-dimensional figures have area, while threedimensional figures have volume and surface area.

A 20-gallon aquarium can measure 24 inches wide, 12 inches deep, and 16 inches high. What is the area of the bottom of the aquarium?







Cut out the correct Foldable from the FL pages in the back of this book.



Place your Foldable on the Key Concept page toward the end of this chapter.



Use the Foldable throughout this chapter to help you learn about volume and surface area.



Vocab Vocabulary

base	slant height
cubic units	surface area
lateral face	three-dimensional figure
prism	triangular prism
pyramid	vertex
rectangular prism	volume

Review Vocabulary

Using a graphic organizer can help you to remember important vocabulary terms. Fill in the graphic organizer below for the phrase *two-dimensional figure*.

{	two-dimensional figure	}
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	Real-World Examples	
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	Drawings	{
		\
		\
	The number of square units needed to cover the surface of a closed figure is	
	the	







Inquiry Lab

Volume of Rectangular Prisms



Work with a partner. Use 36 cubes. Build all the possible prisms with a volume of 36 cubic units. List the dimensions below. Use each set of factors only once.



×	×	= 36
×	×	= 36
×	×	= 36
×	×	= 36

(q)

Investigation 2

You can find the volume of rectangular prisms with fractional side lengths.



The model to the right is _____ cubes long,

_____ cube wide, and _____ cube tall.



Count the number of cubes used to build the model.

The model uses _____ cubes.

So, the volume of the model is _____ cubic feet.

Compare the product of the dimensions of the prism with its volume.



They are _____

Investigation 3

You can use cubes of candy to find the volume of rectangular prisms with fractional sides.



Cut one piece of candy into two halves.

- **Step 2** Make a model that is $2\frac{1}{2}$ cubes long, 2 cubes wide, and 1 cube tall. Draw a picture of your model.
- Step 3
 Count the number of cubes used to build the model. The model uses ______ whole cubes and ______ half-cubes.

 Two halves equal one whole. So, a total of ______ cubes were used.

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So, the volume of the prism is _____ cubic units.

Compare the product of the dimensions of the prism with its volume.

_____ × _____ × ____ = ____

They are _____.

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Collaborate

Model with Mathematics Work with a partner. Use models to determine the volume of each prism. Draw a diagram of each model in the space provided.

- 1. length: 1 height: 1 width: 1 volume:
 - 3. length: 3 height: 4 width: 2 volume:
 - 5. length: $2\frac{1}{2}$

width: 1	F		
width. I			
volume:			
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2. length: 2 height: 4 width: 1 volume:

- 4. length: $\frac{1}{2}$ height: 1 width: 1 volume:
- 6. length: $3\frac{1}{2}$
 - height: 2 width: 2 volume:

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Work with a partner to complete the table. Use models, if needed. The first one is done for you.

	Prism	Height	Length	Width	Volume	
	A	6	3	2	36	
7.	ß	$2\frac{1}{2}$	$1\frac{1}{2}$	2		
8.	С	5	$1\frac{1}{2}$	2		
9.	D	2	5	$1\frac{1}{2}$		
10.	Е	5	3	4		

- **11.** Compare the dimensions for prism *C* to the dimensions of prism *D*. Compare the volume of the two prisms. What do you notice?
- **12.** The length and width of prisms *B* and *C* are equal. Compare the height of the two prisms. How does the change in height affect the change in volume?
- **13.** Compare the dimensions for prism *B* to the dimensions of prism *E*. Compare the volume of the two prisms. What do you notice?
- **14. Base Reason Inductively** Describe the relationship between the number of cubes needed and the dimensions of the prism.



15. Model with Mathematics Write a real-world problem that involves volume of rectangular prisms. Include the dimensions and the volume of the

rectangular prism in your response.



HOW can you use models to find volume?

Lesson 1

Volume of Rectangular Prisms

Vocab

а_{bc}

What You'll Learn

Scan the lesson. Predict two things you will learn about finding the volume of rectangular prisms.

Vocabulary Start-Up





HOW is shape important when measuring a figure?



three-dimensional figure prism rectangular prism volume cubic units



Content Standards MCC6.G.2

Mathematical Practices 1, 3, 4, 5, 6, 7



Aquarium The dimensions of an aquarium are shown.

1. What is the area of the base of the aquarium?



Watch

- 2. What is the height of the aquarium?
- **3.** Fill in the blanks to find the volume.



Work Zone

Volume of a Rectangular Prism



Symbols

Model

h l l

A three-dimensional figure has

length, width, and height. A **prism** is a three-dimensional figure with two parallel bases that are congruent polygons. In a **rectangular prism**, the bases are congruent rectangles.

 $V = \ell wh$ or V = Bh

Volume is the amount of space inside a three-dimensional figure. It is measured in **cubic units**, which can be written using abbreviations and an exponent of 3, such as units³ or in³.

related to its dimensions, length, width, and height.



Cubes

Cubes are special rectangular prisms. All three side lengths are equal. So, the volume of a cube can be written using the formula $V = s^3$.

Decomposing the prism tells you the number of cubes of a given size it will take to fill the prism. The volume of a rectangular prism is

Another method to decompose a rectangular prism is to find the area of the base (B) and multiply it by the height (h).





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Find Missing Dimensions

To find missing dimensions of a rectangular prism, replace the variables with known measurements. Then solve for the unknown measurement.



Guided Practice

d.

Sh

1. A rectangular kitchen sink is 25.25 inches long, 19.75 inches wide, and 10 inches deep. Find the amount of water that can be contained in the

 sink. (Examples 1 and 2) 2. Find the missing dimension volume of 126 cubic cent 	on of a rectangular prism with a imeters, a width of 7 <mark>7</mark> centimeter	ſS,
and a height of 2 centime	eters. (Example 3)	Rate Yourself!
3. Building on the Essen either the formula $V = \ell w$ of a rectangular prism?	ntial Question Why can you use vh or $V = Bh$ to find the volume	Are you ready to move on? Shade the section that applies. YES ? NO
		For more help, go online to access a Personal Tutor.
		FOLDABLES Time to update your Foldable!

 \checkmark

 $12\frac{3}{8}$ yd

10 vd

Independent Practice

Find the volume of each prism. (Example 1)



4. A fishing tackle box is 13 inches long, 6 inches wide, and $2\frac{1}{2}$ inches high. What is the volume of the tackle box? (Example 2)



Go online for Step-by-Step Solutions

eHelp

 Find the length of a rectangular prism having a volume of 2,830.5 cubic meters, width of 18.5 meters, and height of 9 meters. (Example 3)

Find the missing dimension of each prism. (Example 3)



8. Be Precise In Japan, farmers have created watermelons in the shape of rectangular prisms. Find the volume of a prism-shaped watermelon in cubic inches if its length is 10 inches, its width is $\frac{2}{3}$ foot, and its height is 9 inches.

19 The glass container shown is filled to a height of 2.25 inches.

- a. How much sand is currently in the container?
- b. How much more sand could the container hold before it overflows?
- c. What percent of the container is filled with sand? _



10. We ldentify Structure Refer to the graphic novel frame below for Exercises a-c.



- a. What is the volume of the short box Pilar chose on the left?
- b. What is the volume of Amanda's tall popcorn box on the right?
- c. Who received more popcorn, Pilar or Amanda? How much more?

H.O.T. Problems Higher Order Thinking

- **11.** Persevere with Problems Refer to the prism at the right. If all the dimensions of the prism doubled, would the volume double? Explain your reasoning.
- **12. W Justify Conclusions** Which has the greater volume: a prism with a length of 5 inches, a width of 4 inches, and a height of 10 inches, or a prism with a length of 10 inches, a width of 5 inches, and a height of 4

D 1,710 in³

inches? Justify your selection.



- **13.** Don used the shoebox to create a home for the toad he caught. Find the volume of the shoebox.
 - (Å) 222 in³ (C) 1,620 in³
 - B 864 in³

10 in. 18 in.



Extra Practice

Find the volume of each prism.







- **17.** Find the volume of the pet carrier shown at the right.
- 18. What is the width of a rectangular prism with a length of 13 feet, volume of 11,232 cubic feet, and height of 36 feet?



19. The Palo Duro Canyon is 120 miles long, as much as 20 miles wide, and has a maximum depth of more than 0.15 mile. What is the approximate volume of this canyon?

20. 😻 Use Math Tools Use the table at the right.

- a. What is the approximate volume of the small truck?
- **b.** The Davis family is moving, and they estimate that they will need a truck with about 1,250 cubic feet. Which truck would be best for them to rent?
- **c.** About how many cubic feet greater is the volume of the Mega Moving Truck than the 2-bedroom moving truck?

Inside Dimensions of Moving Trucks					
Truck	Length (ft)	Width (ft)	Height (ft)		
Van	10	$6\frac{1}{2}$	6		
Small Truck	$11\frac{1}{13}$	$7\frac{5}{12}$	$6\frac{3}{4}$		
2-Bedroom Moving Truck	$14\frac{1}{2}$	$7\frac{7}{12}$	$7\frac{1}{6}$		
3-Bedroom Moving Truck	$20\frac{5}{6}$	$7\frac{1}{2}$	$8\frac{1}{12}$		
Mega Moving Truck	$22\frac{1}{4}$	$7\frac{7}{12}$	$8\frac{5}{12}$		

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Georgia Test Practice

21. Poppy's Pasta comes in the rectangular box shown. What is the volume of the box?



- ▲ 41.9 cm³
- B 93.16 cm³
- © 1,993.624 cm³
- D 2,058 cm³

- 22. A pet carrier company is creating a new size carrier. It has a width of 27 centimeters, a length of 7 centimeters, and a volume of 6,426 cubic centimeters. Find the height.
 - ① 34 centimeters
 - G 38 centimeters
 - (H) 42 centimeters
 - ① 46 centimeters



23. Short Response The volume of the rectangular prism shown is 2,520 cubic inches. Find the width of the prism.



common Core Review

Classify each triangle by the measure of the angles. MCC5.G.4





27. Draw the next figure in the pattern below. MCC4.OA.5, MCC4.G.2



28. Triangles are often used in designing bridges. Classify the triangle shown by the measure of its sides. Explain. MCC5.G.4



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Lesson 2 Volume of Triangular Prisms

Watch

_8 ft

What You'll Learn

Scan the lesson. Predict two things you will learn about finding the volume of triangular prisms.



HOW is shape important when measuring a figure?



triangular prism



Content Standards Extension of MCC6.G.2

Mathematical Practices 1, 3, 4, 6, 8

Real-World Link

Camping Ari has a pup tent like the one shown. The opening of the tent has a base and a height of 6 feet. The length of the tent is 8 feet.

What is the area of the front

triangular face?



Collaborate On a piece of grid paper, draw a right triangle with a base and height of 4 units as shown.

- **1.** What is the area of the triangle?
- **2.** Suppose you cover the triangle with cubes the size of one square on the grid paper. How many cubes would you use? (*Hint*: You can

6 ft -

cut and reassemble the cubes.)

- 3. How many cubes would you use if you had 4 layers?
- Make a Conjecture Write a formula to find the volume of a triangular prism.



In a triangular prism, the bases are congruent triangles. The diagram below shows that the volume of a triangular prism is also the product of the area of the base B and the height h of the prism.



Example



The volume is 520 cubic meters or 520 m^3 .



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Tutor

Chapter 9 Volume and Surface Area 654

Base

a. .

Ь.

Before finding the volume

Exercise b, the base is not on

the "bottom." The base is

one of the parallel faces.

of a triangular prism,

identify the base. In







4. Dwane bought a cheese wedge for his March Madness party. The cheese wedge has the dimensions shown. The volume of the cheese wedge is 54 cubic inches. What is the height of the cheese wedge?



So, the height of the cheese wedge is 9 inches.

Guided Practice

Find the volume of each prism. Round to the nearest tenth if necessary. (Example 1)

1. 3 ft 5 ft Show your your work of ft



3. Dirk has a triangular-shaped piece of cheesecake in his lunch. Find the volume of the piece of cheesecake. (Example 2)



4 in.

Tutor

3 in.

- 4. Find the base length of a shipping box in the shape of a triangular prism. The shipping box has a volume of 276 cubic feet, a base height of 6.9 feet, and a height of 10 feet. (Examples 3 and 4)
- 5. **Q** Building on the Essential Question How is the area of a triangle related to the volume of a triangular prism?

Rate Yourself!

How well do you understand volume of triangular prisms? Circle the image that applies.



Independent Practice

Go online for Step-by-Step Solutions

Find the volume of each prism. Round to the nearest tenth if necessary. (Example 1)



- A wheelchair ramp is in the shape of a triangular prism. It has a base area of 37.4 square yards and a height of 5 yards. Find the volume of the ramp. (Example 2)
- A triangular prism has a height of 9 inches. The triangular base has a base of 3 inches and a height of 8 inches. Find the volume of the prism. (Example 2)

Find the missing dimension of each triangular prism. (Example 3)



9. Mr. Standford's greenhouse has the dimensions shown. The volume of the greenhouse is 90 cubic yards. Find the missing dimension of the greenhouse. (Example 4)

10. W Be Precise Darcy built the dollhouse shown.

- a. What is the volume of the first floor?
- b. What is the volume of the attic space?



h



Extra Practice

Find the volume of each prism. Round to the nearest tenth if necessary.







- 18. A candle is in the shape of a triangular prism. The base has an area of 30 square inches. The candle has a height of 6 inches. Find the volume of the candle.
- 19. A cabinet is in the shape of a triangular prism. The triangular base has a base length of 14 inches and a base height of 22 inches. The cabinet is 67.5 inches tall. What is the volume of the cabinet?

22. *x* =

Find the missing dimension of each triangular prism.

23. What is the volume of the A-frame tent shown?

dimensions shown.

tenth?

the nearest tenth?

24. We Be Precise A covered bridge in Vermont has the

a. What is the volume of the bottom section rounded to

b. What is the volume of the roof, rounded to the nearest









Georgia Test Practice

25. Find the volume of the triangular prism shown, rounded to the nearest tenth.



- **26.** A triangular prism has a volume of 240 cubic meters. Which of the measurements below are *not* possible dimensions for the area of the base and the height of the prism?
 - (F) $B = 48 \text{ m}^2$, h = 5 m
 - G $B = 24 \text{ m}^2$, h = 10 m
 - (H) $B = 12 \text{ m}^2$, h = 20 m
 - ① $B = 20 \text{ m}^2$, h = 4 m
- **27. Short Response** Tia wants to purchase the corner kitchen cabinet with the greater volume. Find the volume of each cabinet to determine which one Tia should buy.



🥵 Common Core Review

Find the area of each figure. MCC4.MD.3

	_	

28.

23.		_		

-		

31. Sarah is building a birdhouse. The nails she uses are one inch long. The wood board is 1 foot long. How many times smaller are the nails

compared to the wood? MCC4.MD.1

32. The floor area of Melana's rectangular closet is 18 square feet. The closet

is 6 feet long. How wide is the closet? MCC4.MD.3

Problem-Solving Investigation Make a Model

Content Standards MCC6.G.2, MCC6.G.4

1, 3, 4

Mathematical Practices

Case #1 Scooter Storage

Nick works for a sporting goods store. He is stacking boxes of scooters in the storage space at the back of the store. The first laver has 9 boxes.

If the storage area will hold 6 layers of boxes, how many boxes will the storage space hold?

Understand What are the facts?

- The first layer has 9 boxes.
- The storage space will hold 6 layers.





Make a model using centimeter cubes.

Solve How can you apply the strategy?

Make a model of one layer of the box by arranging 9 cubes in a 3 \times 3 array.

Continue stacking the cubes until there are 6 layers.

So, the storage space will hold 54 boxes.



Check Does the answer make sense?

Use the volume formula to check your answer. $V = 3 \times 3 \times 6$ or 54

So, the storage space will hold a total of 54 boxes.

Tools **Analyze the Strategy**

Justify Conclusions Suppose the boxes are a different size and the first layer has 6 boxes instead. How many boxes can be stored if the storage space will hold 5 layers? Explain.

Tutor





Case #2 Contain Your Fun

A storage container is made from plastic that measures $1\frac{1}{2}$ feet long, 2 feet wide, and $2\frac{1}{2}$ feet high. Find the surface area of the plastic container, including the lid.



Understand

Read the problem. What are you being asked to find?

I need to find

Underline key words and values in the problem. What information do you know?

The storage container measures _____ long, _____ wide,

and _____ high.

Plan

Choose a problem-solving strategy.



strategy.

2 ft

 $2\frac{1}{2}$ ft

Solve

Use your problem-solving strategy to solve the problem.

Make a model of the container using a net. Then find the area of each rectangle to find the total surface area.

front and back: 2(×) =	
left and right: 2(×) =	
top and bottom: 2(×) =	
Sum of 6 sides:	+	+	
	T	T	

So, the surface area of the container is ______ square feet.



Check

Use information from the problem to check your answer.

Substitute known values into the surface area formula to check your answer.

S.A. = (_____) + (_____) + (_____) = ____ft^2

, Inc



Collaborate Work with a small group to solve the following cases. Show your work on a separate piece of paper.

Case #3 Assembly

DJ is helping set up 7 rows of chairs for a school assembly. There are eight chairs in the first row. Each row after that has two more chairs than the previous row.

If he has 100 chairs, can he set up enough rows? Explain.

Watch

Case #4 Paper

Timothy took a piece of notebook paper and cut it in half. Then he placed the 2 pieces on top of each other and cut them in half again to have 4 pieces of paper.

If he could keep cutting the paper in this manner, how many pieces of paper would he have after 6 cuts?

Case #5 Sports

Rosario is packing a crate with boxes of miniature golf putters. Each box has a height of 1 foot, a width of 1 foot, and a length of 3 feet.

1 ft 3 ft

How many boxes can Rosario fit in a crate that is 4 feet high, 4 feet wide, and 3 feet long?

Circle a strategy below to solve the problem. • Look for a pattern.

- · Actitout.
- · Work backward.
- · Guess, check and revise.

Case #6 Patterns

Draw the seventeenth figure in the pattern.





Mid-Chapter Check

Vocabulary Check



1. Be Precise Define three-dimensional figure. Give an example of a figure that is a three-dimensional figure and an example of a figure that is not a three-dimensional figure. (Lesson 1)

Fill in the blanks in the sentences below with the correct terms. (Lesson 1)

- 2. Volume is the amount of ______ inside a three-dimensional figure.
- 3. Volume is measured in _____ units.

Skills Check and Problem Solving

Find the volume of each prism. Round to the nearest tenth

if necessary. (Lessons 1 and 2)



Find the missing dimension of each figure. (Lessons 1 and 2)

- 7. rectangular prism: $V = 80 \text{ m}^3$; length = 5 m; width = 4 m
 - h = _____

- 8. triangular prism: $V = 42 \text{ cm}^3$; base length = 2 cm; base height = 6 cm h =_____
- **9.** A storage unit is in the shape of a rectangular prism. Find the volume of the storage unit. (Lesson 1)



10. Georgia Test Practice What is the height of a rectangular prism with a volume of 63 cubic feet and a base area of 15 square feet? (Lesson 1)

- (A) 945 feet
 (C) 48 feet
- B 78 feet
 D 4.2 feet

Inquiry Lab

Surface Area of Rectangular Prisms



Content Standards MCC6.G.4

> Mathematical Practices 1, 3, 4

Cereal If you want to know the amount of cereal that can fit in the box, you would find the volume. But if you want to know how much cardboard is needed to make the box, you would find the *surface area*.

Investigation 1

One way to find the surface area is to use a *net*. Nets are two-dimensional patterns of three-dimensional figures. When you construct a net, you are decomposing the three-dimensional figure into separate shapes.



Step 1

Use a cereal box in the shape of a rectangular prism. Measure and record the length, width, and height of the box on the lines below.

Length:

Width:

Height:



Step 2

Using a marker, label the top, bottom, front, back and side faces of the box.



3 Using scissors, carefully cut along three edges of the top face and then cut down each vertical edge.

Step 4 Measure and record the area of each face, using the dimensions of the box shown in the table.



Add the areas of each face to find the surface area of the box.

Face	Length	Width	Area of Face	
Front			2	
Back				
Side 1				
Side 2				
Тор				
Bottom				



So, the surface area of the box is

square inches.

Investigation 2

Orthogonal drawings consist of separate views of an object taken from different angles. You can make a net from orthogonal drawings.



Find the dimensions of each side of a rectangular prism from the orthogonal drawing.

Orthogonal Drawing				
View	Drawing	Dimensions		
Front and Back		×		
Sides		×		
Top and Bottom		×		

Step 2

Use grid paper to draw a net from the orthogonal drawing. Trace and cut out your drawing and tape it in the space below. Check the dimensions of each face using the information in the table.





square units.

So, the figure is a _____.

It has a surface area of

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Collaborate

Model with Mathematics Work with a partner. Use a net to determine the surface area of each prism. Draw a net of each prism on the provided grid.



Draw a net on the grid from the orthogonal drawing. Then find the surface area of the prism.

5. Square units

 Square units

 Orthogonal Drawing

 View
 Drawing

 Front and Back
 Image: Colspan="2">Image: Colspan="2">Image: Colspan="2">Image: Colspan="2">Image: Colspan="2">Image: Colspan="2">Image: Colspan="2">Image: Colspan="2"

 Sides
 Image: Colspan="2">Image: Colspan="2"

 Top and Bottom
 Image: Colspan="2">Image: Colspan="2"





Work with a partner to complete the table. The first one is done for you.

	Dimensions of Rectangular Prism	Area of Top (units ²)	Area of Bottom (units ²)	Area of Side 1 (units ²)	Area of Side 2 (units ²)	Area of Front (units ²)	Area of Back (units ²)	Surface Area (units ²)	
	$1 \times 2 \times 3$	2	2	6	6	3	3	22	
6.	$2 \times 2 \times 3$							-	
7.	$3 \times 3 \times 3$								
8.	3 × 2 × 8								
9.	6 × 6 × 6								

10. Compare the surface area for Exercise 7 to the surface area for Exercise 9. How does doubling each dimension affect the surface area?

11. Reason Inductively Write a formula to find the surface area of a rectangular prism. Use your formula to find the surface area of the prism

in Investigation 2.



12. Model with Mathematics Write a real-world problem that involves the surface area of rectangular prisms. Provide the dimensions and the surface

area.

13. Will the surface area of a cube ever have the same numerical value as the volume of the cube?



(Main) HOW can you use nets to find surface area?

Lesson 3

Surface Area of Rectangular Prisms

abc

What You'll Learn

Scan the lesson. Predict two things you will learn about finding the surface area of rectangular prisms.

HOW is shape important when





surface area



Content Standards MCC6.G.4

Mathematical Practices 1, 3, 4, 8

Vocabulary Start-Up

Define Surface	Define Area
What is surface area?	Example:



Gifts Roberta is wrapping a gift for her sister's quinceañera. She places it in a box with the measurements shown below.

1. What is the area of one face of the box?



- 2. How many faces does the box have?
- 3. What operations would you use to find the surface area of the box?

Key Concept

Work Zone

Surface Area of a Rectangular Prism

Words The surface area S.A. of a rectangular prism with length ℓ , width *w*, and height *h* is the sum of the areas of the faces.

Model

h w l

Symbols S.A. = $2\ell h + 2\ell w + 2hw$

The surface area of a prism is the sum of the areas of its faces.



front and back: $\ell h + \ell h = 2\ell h$ top and bottom: $\ell w + \ell w = 2\ell w$ two sides: hw + hw = 2hw

 $\left. \begin{array}{c} 2\ell h+2\ell w+2hw \end{array} \right\} 2\ell h+2\ell w+2hw$

6 m

7 m 42 m²

Example

1. Find the surface area of the rectangular prism.

Find the area of each pair of faces. front and back: $2(8 \cdot 6) = 2(48)$ top and bottom: $2(7 \cdot 8) = 2(56)$ sides: $2(7 \cdot 6) = 2(42)$

48 + 48 + 56 + 56 + 42 + 42 = 292

2 Add the area of each face.

8 m

48 m²

56 m²

48 m²

6 m

6 m

42 m²

6 m

So, the surface area is 292 square meters.

Got It? Do this problem to find out.

 a. Find the surface area of the rectangular prism.



Nets

a. .

The net shows that a rectangular prism has six faces. The faces can be grouped as three pairs of congruent sides. The colors indicate which faces are congruent.

Show



8 m

56 m²

Find Surface Area Using a Formula

You can use nets or models to find the surface area of a rectangular prism. You can also use the surface area formula, S.A. = $2\ell h + 2\ell w + 2hw$.

Tutor **Examples** 2. Find the surface area of the 4 ft rectangular prism. 5 ft Find the area of each face. 7 ft front and back: 5 ft $2\ell h = 2(7)(4)$ or 56 7 ft top top and bottom: 7 ft 7 ft 5 ft $2\ell w = 2(7)(5)$ or 70 4 ft 4 ft back side front side left and right sides: 2hw = 2(4)(5) or 40 bottom 7 ft Add to find the surface area. The surface area is 56 + 70 + 405 ft or 166 square feet. **3.** Find the surface area of the rectangular prism. 6 cm To find the area of each face determine 4.8 cm the dimensions. 7 cm $\ell = 7, w = 4.8, h = 6$ front and back: $2\ell h = 2$ or top and bottom: $2\ell w = 2^{1/2}$ two sides: 2hw = 2or Add to find the surface area. square centimeters or Show Got It? Do this problem to find out. Work Ь. **b.** Find the surface area of the rectangular prism. 4 m 3 m 2 m





Example

4. STEM A geode is being sent as a gift. It is packed in a box that measures 7 inches long, 3 inches wide, and 16 inches tall. What is the surface area of the box?

Tutor

Check

1

$S.A. = 2\ell h + 2\ell w + 2hw$	Surface area of a prism
S.A. $= 2(7)(16) + 2(7)(3) + 2(16)(3)$	$\ell=7, w=3, h=16$
S.A. = 14(16) + 14(3) + 32(3)	Multiply.
S.A. = 224 + 42 + 96	Multiply.
S.A. = 362	Add.

The surface area of the box is 362 square inches.

Guided Practice





Independent Practice

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eHelp





- 5. **STEM** A game box for video games is shaped like a rectangular prism. What is the surface area of the game box? (Example 4)
- 6. We Justify Conclusions Martina estimates that the surface area of a rectangular prism with a length of 13.2 feet, a width of 6 feet, and a height of 8 feet is about 460 square feet. Is her estimate reasonable? Explain your reasoning.



Which package has the greater surface area? Does the same package have a greater volume? Explain your reasoning to a classmate.



8. (WF) Model with Mathematics Refer to the graphic novel frame below for Exercises a–c. (*Hint:* The boxes are missing the top face.)



- **b.** What is the surface area of the tall box on the right?
- c. How much more surface area does the larger container have?

H.O.T. Problems Higher Order Thinking

Persevere with Problems All of the triangular faces of the figure are congruent.

- 9. What is the area of one of the triangular faces? the square face?
 - of a rectangular 12 in.
- **10.** Use what you know about finding the surface area of a rectangular prism to find the surface area of the square pyramid.

Georgia Test Practice

11. Which net can be used to make and find the surface area of a cube?



8 in.

12 in.

Name

Extra Practice

Find the surface area of each rectangular prism.



3 ft



16. Nadine is going to paint her younger sister's toy chest, including the bottom. What is the approximate surface area

that she will paint? _

7 ft

5 ft

- 17. Identify Repeated Reasoning Chrissy is making a bird nesting box for her backyard.
 - a. What is the surface area of the nesting box, including

the hole?

b. What is the surface area if the width is doubled?



/ 19 in.

19 in.



7.5 in.

Georgia Test Practice

- **18.** Which measure can be classified as surface area?
 - (A) the amount of water in a lake
 - B the amount of land available to build a house
 - © the amount of wrapping paper needed to cover a box
 - D the height of a tree

19. Dilip is going to paint a shoebox to use for storage of his trading cards. The shoebox is 23 inches long, 10 inches wide, and 8 inches high. Find the surface area of the shoebox.

12 in.

- 246 square inches
- 828 square inches
- (H) 988 square inches
- ① 1,840 square inches
- **20. Short Response** A company is experimenting with two new boxes for packaging merchandise. Each box is a cube with the side lengths shown. What is the ratio of the surface area of the smaller box to the surface area of the larger box?



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Inquiry Lab

Nets of Triangular Prisms

Inquiry

HOW is the area of a triangle related to the surface area of a triangular prism?

Content Standards MCC6.G.4

> Mathematical Practices 1, 3, 4, 7

Packaging A computer hardware company packages batteries and cords in boxes shaped like triangular prisms. You can use nets and drawings to determine the surface area of the box.

Investigation

Use orthogonal drawings to find the surface area of a triangular prism. A *triangular prism* is a prism that has triangular bases.



Step 1

Find the dimensions of each side of th	ne triangular prism from
the orthogonal drawing.	

Orthogonal Drawing							
View	Drawing	Dimensions (in.)	Area of Face (in ²)	View	Drawing	Dimensions (in.)	Area of Face (in ²)
Bases		base = 3 height = 4	$\frac{1}{2}(3\times 4)=6$	Bottom		length $= 6$ width $= 5$	6 × 5 = 30
Left		length = 6 width = 3	6 × 3 = 18	Right		length = 6 width = 4	6 × 4 = 24

Step 2

Use grid paper to draw a net. Check the dimensions of each face using the information in the table.



Add the area of each face to find the surface area of the figure. Remember, there are two bases.





Model with Mathematics Work with a partner. Use nets to determine the surface area of each prism. Draw a net of each prism on the provided grid paper.





3. Identify Structure Explain how to find the surface area of a triangular prism, using only the dimensions of the figure. Use the dimensions in Exercise 2 to explain your answer.

4. Inquiry

HOW is the area of a triangle related to the surface area of a triangular prism?

Lesson 4

Essential Question

Surface Area of Triangular Prisms

What You'll Learn

Scan the lesson. Predict two things you will learn about finding the surface area of triangular prisms.



Mathematical Practices 1, 2, 3, 4, 6

Real-World Link

Ramp Raj and his dad are building a ramp to move his dirt bike onto a trailer.



Fill in the table by drawing the sides of the ramp and naming the shape of each face.



Key Concept

Work Zone

Surface Area of a Triangular Prism

Words

The surface area of a triangular prism is the sum of the areas of the two triangular bases and the three rectangular faces.



A triangular prism is a prism that has triangular bases. When the bases are equilateral triangles, the areas of the three rectangular faces are equal. You can use a net to find the surface area of a triangular prism.

Example

1. Find the surface area of the triangular prism.

To find the surface area of the triangular prism, find the area of each face and add.



1 cm

Tutor

0.8 cm 2 cm 1 cm

0.8 cm

1 cm

area of each triangular base: $\frac{1}{2}(1)(0.8) = 0.4$ area of each rectangular face: 1(2) = 2Add to find the surface area.

0.4 + 0.4 + 2 + 2 + 2 = 6.8 square centimeters

1 cm

Got It? Do this problem to find out.

a. Find the surface area of the triangular prism.



a.

Surface Area of Other Triangular Prisms

You can also find the surface area of any triangular prism by adding the areas of all the sides of the prism using an orthogonal drawing.





e

Independent Practice

7.5 in.

Find the surface area of each triangular prism. (Examples 1–2)



5 in.

- **8. We multiple Representations** The figure shows the dimensions of a triangular prism.
 - a. Models Draw a model of the faces and bases of the triangular prism.



- **b. Words** Describe the triangular prism.
- c. Numbers Find the surface area of the triangular prism using addition.
- **9.** The surface area of a right triangular prism is 228 square inches. The base is a right triangle with a base height of 6 inches and a base length of 8 inches. The length of the third side of the base is 10 inches. Find the

height of the prism.

H.O.T. Problems Higher Order Thinking

- **10.** Reason Abstractly Describe the dimensions of a triangular prism that has a surface area between 550 square inches and 700 square inches.
- **11. Persevere with Problems** Sketch and label two triangular prisms such that one has a greater volume and the other has a greater surface area.

Georgia Test Practice

12. Find the surface area of the prism to the nearest whole number.

- (A) $1,471 \text{ cm}^2$
- B 945 cm²
- © 1,007 cm²
- D 777 cm²



Extra Practice





Copy and Solve Find the surface area of each triangular prism using the base triangles shown. Show your work on a separate piece of paper.



Georgia Test Practice

21. What is the surface area of the prism shown below?



22. Short Response A triangular prism has bases that each have an approximate area of 173 square centimeters. The triangles measure 20 centimeters on each side and the prism is 4 centimeters high. What is the surface area of the prism?

23. The attic of the house is a wooden surface. How many square meters of wood are needed to build the roof and floor of the attic?



① 640 m²

🚱 Common Core Review

Identify each triangle as acute, right, or obtuse. MCC5.G.4







- **27.** A certain two-dimensional figure has two pairs of parallel lines, four right angles, and four congruent sides. What is the figure? MCC4.G.2
- **28.** The showcase floor at Murphy Motors is divided into four equal sections. Find the area of Motorcycle Madness if the square showcase floor has a side length of 20 feet. Mcc4.MD.3



Inquiry Lab Nets of Pyramids



HOW is the area of a triangle related to the surface area of a square pyramid?

Content Standards MCC6.G.4

> Mathematical Practices 1, 3, 4

Art Anderson Art is designing a paper weight that is shaped like a square pyramid.

Investigation

Use orthogonal drawings to find the surface area of a square pyramid. A *square pyramid* is a three-dimensional figure with a square base and four triangular faces.



Step 1

Find the dimensions of each side of the square pyramid from the orthogonal drawing.

Orthogonal Drawing						
View	Drawing		Dimensions (ft)	Area of Face (ft ²)		
Bases	4 ft	4 ft		length = 4 height = 4	4 × 4 = 16	
Triangular Faces	3 ft 4 ft		height = 3 base = 4	$\frac{1}{2}(3 \times 4) = 6$		



Use grid paper to draw a net. Check the dimensions of each face using the information in the table.

Step 3

+

Add the area of each face to find the surface area of the figure. Remember, there are four triangular faces.



So, the surface area is

Х

square units.

Collaborate

Model with Mathematics Work with a partner. Use nets to determine the surface area of each pyramid. Draw a net of each pyramid on the provided grid paper.





3. Construct an Argument Explain how to find the surface area of a square pyramid, without creating a net. Use the dimensions in Exercise 1 to explain your answer.

4. Inquiry

HOW is the area of a triangle related to the surface area of a

square pyramid? _

Lesson 5 Surface Area of Pyramids

Vocab

abc

What You'll Learn

Scan the lesson. Predict two things you will learn about finding the surface area of pyramids.

Vocabulary Start-Up

A **pyramid** is a three-dimensional figure with at least three triangular sides that meet at a common vertex and only one base that is a polygon. The triangular sides of a square pyramid are called the lateral faces. The slant height is the height of each lateral face.

Fill in the blanks on the diagram below with vocabulary words.



lateral face lateral face lateral face lateral face



Museum Claude made a model of the large pyramid in front of the Louvre museum. His model is shown.

1. Draw the faces of the pyramid.



5 in.



Essential Question

Common Core GPS

HOW is shape important when

Vocabulary

measuring a figure?

Vocab

ub_c

FFEIS

MCC6.G.4

pyramid vertex base lateral face

slant height

Content Standards

base

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lnc.

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Key Concept

Work Zone

Surface Area of a Pyramid





Some pyramids have square or rectangular bases. You can use a net to find the surface area of a pyramid.



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a.

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Surface Area of Pyramids with Triangular Bases

A triangular pyramid has one triangular base, and three triangular faces. If the base is an equilateral triangle, all three lateral faces are congruent. If the sides of the base triangle are different lengths, the areas of the lateral faces will also vary.





Example



Check

ſ

3. A pyramid puzzle has all sides that are equilateral triangles. Each triangle has side lengths of 8 centimeters. The slant height is 6.9 centimeters. Find the surface area of the puzzle.

Create a net and then use it to find the surface area of the pyramid.

Each face has an area of $\frac{1}{2}(8)(6.9)$ or 27.6 square centimeters. So, the surface area of the puzzle is $4 \cdot 27.6$ or 110.4 square centimeters.

6.9 cm 6.9 cm 8 cm 8 cm 6.9 cm

Guided Practice

Find the surface area of each pyramid. (Examples 1–2)



Independent Practice

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eHelp

Find the surface area of each pyramid. (Examples 1–2)



- A tea bag is shaped like a square pyramid with the base measuring 4 centimeters on each side. The slant height is 4.5 centimeters. How much mesh is used to create the tea bag? (Example 3)
- 8. An earring design is shaped like a triangular pyramid. All the faces are equilateral triangles with side lengths of 14 millimeters. The slant height is 12.1 millimeters. What is the surface area of the earring? (Example 3)
- An acting award is a square pyramid with a base that measures 6 inches on each side. The slant height is 8 inches. What is the surface area of the award? (Example 3)

10. (B) Identify Structure Refer to the figures listed in the table. Determine the number of faces the figure has of each two-dimensional shape. Explain.

Figure	Rectangular Faces	Triangular Faces
Rectangular Prism		
Triangular Prism		
Square Pyramid		
Triangular Pyramid		



Extra Practice

Find the surface area of each pyramid.



20. A paper model of the Khafre pyramid in Egypt has a square base 7.2 centimeters on each side. The slant height is 6 centimeters. How much paper was used to make the model?



21. Be Precise A triangular pyramid has a surface area of 336 square inches. It is made up of equilateral triangles with side lengths of 12 inches. What is the slant height?



Georgia Test Practice



23. Short Response A pyramid has a base that is an equilateral triangle. The area of the base is 209.6 square inches and the sides measure 22 inches. The slant height of the pyramid is 14 inches. What is the surface area?

24. A salt shaker is in the shape of a square pyramid. The net is shown below. What is the surface area of the salt shaker?



- G 40.8 cm²
- 🕀 81.6 cm²
- ① 97.6 cm²





Interior Designer

Do you like coming up with new ways to decorate your room, or are you always rearranging the furniture? You could have a career doing just that by becoming an interior designer. Interior designers plan the interior space and furnishings of homes, offices, and other places. Their designs are based on the client's specifications, tastes, and budget. Interior designers are responsible for recommending color schemes, furniture, lighting, and remodeling options. Many interior designers also develop their own product lines such as furniture, bedding, and accessories.





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Is This the Career for You?

Are you interested in a career as an interior designer? Take some of the following courses in high school.

- Algebra
- Geometry
- Interior Design
- Intro to CAD

Turn the page to find out how math relates to a career in Design.



You be the Designer!

Use the labeled figures to solve each problem. Round to the nearest tenth if necessary.

 A client wants to buy the rectangular ottoman with the most storage area inside. Which one should she choose? Explain your

reasoning.

- 2. Find the volume of the paisley blanket chest.
- **3.** What is the volume of the toy chest? How does it compare to the volume of the



- **4.** A designer is having the red ottoman reupholstered. Find the surface area to estimate the amount of fabric needed.
- **5.** What is the surface area of the purple ottoman?
- **6.** How much greater is the surface area of the paisley blanket chest than the surface

area of the toy chest?





34 in

9.4 in.

Career Project

It's time to update your career portfolio! Use grid paper to make a scale drawing of a room in your home. Model the furniture using squares, rectangles, and triangles drawn to scale. Cut out each shape and use them to create different room arrangements. Then, tape the pieces onto the grid paper. Describe the room's color scheme and style. Do you think you would enjoy a career as an interior designer? Why or why not?

Vocabulary Check

Complete each sentence using the vocabulary list at the beginning of the chapter. Then circle the word that completes the sentence in the word search.

- **1.** A figure with length, width, and height is
 - а____
- 2. _____ is the sum of the area of all the faces of a three-dimensional figure.
- 3. The amount of space inside a three-

dimensional figure is its

- 4. A prism that has triangular bases is
 - a _____
- **5.** A _____ is a prism that has rectangular bases.

- 6. Volume is measured in ____
- The point where three or more faces intersect is the
- 8. The ______ is the height of each lateral face.
- **9.** Any face that is not a base is

а

V R O D I S X W A J A K S M K H N Y P K F Т V TNGBZWG E REAGK G Q J U T C В R G S TOEXL TERA A C X Α F Ε T Ε A Ρ 1 L D Т E GHI EHY L S L BU 1 Т J Т N U C C X A F F GCHHV F V EXK X ZU MHG 0 Т E N F E Ε E E R U G 1 L A N 0 L S Ν M D R G H Т A Κ В E LMAE R A E С A F R U S F U JL K R GUKO U HHU K D K L P Ν Т KQ Y H E E L J F V OMQ V S Т K L N 0 Y S BR Ζ Q A K V R Y S HUW D Ν M В Т Ρ BO TS U L F C R J J SMUD R ECTANG U R Ρ R V С C Ρ L A Y A D Z V G M U L W W Q R I K R A K Н KI D R M TBYUXBSF NSOKEDXGC P Ζ F G 1 N IWYQAZKEDVUOLULHNBJ S S N 0 AHPVJAKOWNZWEIUDQQHVIM



Key Concept Check

Use Your Foldables

Use your Foldable to help review the chapter.

(Tape here)		
Tab 1		
Real-World Examples		
r I		
Tormulas	Model	
Tab 2		

Got it?

Use the figure provided to complete the cross number puzzle.



Across

- 1. surface area of the prism
- 2. height of the base triangle
- 4. height of the prism
- 5. length of the base triangle

Down

13 m

10 m

- 1. area of the base
- 3. volume of the prism
- 4. length of one side of the base triangle

Problem Solving

1. An office building is built in the shape of a rectangular prism. It has a length of 168 yards, a width of 115 yards, and a height 96 yards. What is

the volume	of the	building?	(Lesson 1)
------------	--------	-----------	------------

2. Liza is researching packaging options for her shop. She wants to know the volume of a triangular prism 9 inches tall with a triangular base that is

3 inches by 2 inches. (Lesson 2)

3. We Math Tools Justin is building a storage trunk like the one shown. How much wood is needed to make the trunk? (Lesson 3)



4. A baker needs to put icing on a rectangular cake. The cake is 14 inches long, 12 inches wide, and 4 inches tall. What is the surface area

of the cake, not including the bottom? (Lesson 3) _

does she need to make the tent? (Lesson 4) _

5. Stephanie was making a tent like the one shown. How much fabric

4 ft 3.2 ft 4 ft 4 ft

6. Reason Abstractly A paper die has sides that are all equilateral triangles. Each triangle has a side length of 1.5 cm. The slant height is 1.3 cm. Find the surface area of the die. (Lesson 5)

Reflect



Answering the Essential Question

Use what you learned about volume and surface area to complete the graphic organizer.

	Essent	ial Question	
HOW is shape important when measuring a figure?			
	Draw it.	How do you find the volume?	How do you find the surface area?
rectangular prism			
triangular prism			



Answer the Essential Question. HOW is shape important when measuring a figure?

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